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# Module 1 Topic 1: Quantities and Relationships

Primary Resource: High School Math Solution Algebra I, Carnegie Learning, 2018.

#### **Enduring Understandings**

- Relationships can be described, and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways.
- Mathematical situations and structures can be translated and represented abstractly using variables, expressions, and equations.
- If two quantities vary proportionally, that relationship can be represented as a linear function.
- Mathematical rules (relations) can be used to assign members of one set to members of another set. A special rule (function) assigns each member of one set to a unique member of the other set.

#### **Essential Questions**

• What are the key characteristics of linear, exponential, quadratic, and linear absolute value functions?

Lesson Title	Lesson Overview	Standards
Mathia	By the end of Module 1, Topic 1, students should complete the following units in Mathia:	
	<ul> <li>Understanding Quantities and Their Relationships (1 workspace)</li> </ul>	
	Recognizing Functions and Function Families     (4 workspaces)	



A Picture Is Worth a Thousand Words	Students identify the independent and dependent quantities for various real-world scenarios, match a graph to the scenario, and interpret the scale of the axes. They observe similarities and differences in the graphs, and then focus on key characteristics, such as intercepts, increasing and decreasing intervals, and relative maximum and minimum points.	A.REI.D.10 F.IF.A.1 N.Q.A.1 N.Q.A.2 F.IF.B.4
A Sort of Sorts	Students sort a variety of graphs based on their own rationale, compare their groupings with their classmates', and discuss the reasoning behind their choices. Next, four different groups of graphs are given, and students analyze the groupings and explain possible rationales behind the choices made. Students explore different representations of relations.	F.IF.B.4
F of $X$	Function notation is introduced. The terms <i>increasing function, decreasing function,</i> and <i>constant function</i> are defined. Students sort the graphs from the previous lesson into groups using these terms and match each graph with its appropriate equation written in function notation. The terms <i>function family, linear function,</i> and <i>exponential function</i> are then defined. Next, the terms <i>absolute minimum</i> and <i>absolute maximum</i> are defined. Students sort the remaining graphs into groups using these terms and match each graph with its appropriate equation written in function notation. The terms <i>quadratic function</i> and <i>linear absolute value function</i> are then defined. Linear piecewise functions are defined, and students match the remaining graphs to their appropriate functions. In the final activity, students demonstrate how the families differ with respect to their intercepts.	F.IF.A.1 F.IF.B.4 F.IF.B.5
Function Families for 200, Alex	Given characteristics describing the graphical behavior of specific functions, students name the possible function family/families that fit each description. Students revisit the scenarios and graphs from the first lesson, name the function family associated with each scenario, identify the domain, and describe the graph. Students then write equations and sketch graphs to satisfy a list of characteristics. They conclude by determining that a function or equation, not just a list of characteristics, is required to generate a unique graph.	F.IF.B.4



## **Module 1 Topic 2: Sequences**

Primary Resource: High School Math Solution Algebra I, Carnegie Learning, 2018.

## **Enduring Understandings**

- Relationships can be described, and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways.
- Mathematical situations and structures can be translated and represented abstractly using variables, expressions, and equations.
- If two quantities vary proportionally, that relationship can be represented as a linear function.
- Mathematical rules (relations) can be used to assign members of one set to members of another set. A special rule (function) assigns each member of one set to a unique member of the other set.

#### **Essential Questions**

- What are the advantages and limitations of using tables, functions, and graphs to solve problems?
- What is the relationship between arithmetic sequences and linear functions and geometric sequences and exponential functions?
- What are the effects that horizontal, vertical, and reflective transformations have on exponential functions?

Lesson Title	Lesson Overview	Standards
Mathia	By the end of Module 1, Topic 2, students should complete the following units in Mathia:  • Recognizing Patterns and Sequences (2 workspaces)  • Determining Recursive and Explicit Expressions (2 workspaces)	
Is There a Pattern Here?	Given ten contexts or geometric patterns, students write a numeric sequence to represent each problem. They represent each sequence as a table of values, state whether each sequence is increasing or decreasing, and describe the sequence using a starting value and operation. They determine that all sequences are functions and have a domain that includes only positive integers. Infinite sequence and finite sequence are defined.	F.BF.A.1a F.IF.A.3 F.IF.B.5



The Password Is Operations!	Given 16 numeric sequences, students generate additional terms and describe the rule they used for each sequence. They sort the sequences into groups based upon common characteristics and explain their rationale. The terms arithmetic sequence, common difference, geometric sequence, and common ratio are defined with examples. They then categorize the given sequences based on the definitions and identify the common difference or common ratio where appropriate. Students then practice writing sequences with given characteristics.	F.BF.A.2
Did You Mean: Recursion?	Scenarios are presented that can be represented by arithmetic and geometric sequences. Students determine the value of different terms in each sequence. As the term number increases it becomes more time consuming to generate the term value, which sets the stage for explicit formulas to be defined and used. Students practice using these formulas to determine the values of terms in both arithmetic and geometric sequences	F.BF.A.1a



## **Module 1 Topic 3: Linear Regression**

Primary Resource: High School Math Solution Algebra I, Carnegie Learning, 2018.

## **Enduring Understandings**

- Relationships can be described, and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways.
- Mathematical situations and structures can be translated and represented abstractly using variables, expressions, and equations.
- If two quantities vary proportionally, that relationship can be represented as a linear function.
- Mathematical rules (relations) can be used to assign members of one set to members of another set. A special rule (function) assigns each member of one set to a unique member of the other set.
- Data can be represented visually using tables, charts, and graphs. The type of data determines the best choice of visual representation.

#### **Essential Questions**

- How is a linear regression line of a data set determined?
- How do you determine if a linear model is a good fit for a data set based upon the residual plot?
- What are the advantages and limitations of using tables, functions, and graphs to solve problems?
- Which form of a linear equation is most appropriate to represent a given problem situation?

Lesson Title	Lesson Overview	Standards
Mathia	By the end of Module 1, Topic 3, students should complete the following units in Mathia:  • Least Squares Regression (2 workspaces)  • Correlation (2 workspaces)	
Like a Glove	Students informally approximate a line of best fit for a given data set, write an equation for their line, and then use their function to make predictions, learning about interpolation and extrapolation. They are then introduced to a formal method to determine the linear regression line of a data set via graphing technology.	N.Q.A.3 S.ID.B.6a S.ID.B.6c S.ID.C.7



Gotta Keep It Correlatin	Students analyze graphs and estimate a reasonable correlation coefficient based on visual evidence. They then use technology to determine a linear regression and interpret the correlation coefficient. Next, students analyze several problem situations to determine whether correlation is always connected to causation.	N.Q.A.3 S.ID.B.6a S.ID.B.6c S.ID.C.8 S.ID.C.9
The Residual Effect	Students calculate a linear regression for a real-world problem and analyze the correlation coefficient to conclude whether the linear model is a good fit. The terms residual and residual plot are defined. Students calculate the residuals, construct a residual scatter plot, and conclude by its shape that there may be a more appropriate model. They are given a second data plot. Students create a scatter plot and determine the equation for the least squares regression line and the correlation coefficient with respect to the problem situation. They then calculate residuals and create a residual plot to conclude how the data are related.	S.ID.B.6a S.ID.B.6b S.ID.B.6c
To Fit or Not to Fit? That is the Question!	Students construct a scatter plot, determine a linear regression equation, compute the correlation coefficient, determine the residuals, and create a residual plot for a data set with one variable. They use all the given information to decide whether a linear model is appropriate. A quadratic function is given, and students conclude that this type of function appears to be a better fit. Finally, students summarize how the shape of a scatter plot, the correlation coefficient, and the residual plot help determine whether a linear model is an appropriate fit for the data set. The lesson emphasizes the importance of using more than one measure to determine if a linear model is a good fit.	N.Q.A.3 S.ID.B.6a S.ID.B.6b S.ID.B.6c



## **Module 2 Topic 1: Linear Functions**

Primary Resource: High School Math Solution Algebra I, Carnegie Learning, 2018.

#### **Enduring Understandings**

- Relationships can be described, and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways.
- Mathematical situations and structures can be translated and represented abstractly using variables, expressions, and equations.
- If two quantities vary proportionally, that relationship can be represented as a linear function.
- Mathematical rules (relations) can be used to assign members of one set to members of another set. A special rule (function) assigns each member of one set to a unique member of the other set.

#### **Essential Questions**

- What is the relationship between arithmetic sequences and linear functions and geometric sequences and exponential functions?
- What are the effects that horizontal, vertical, and reflective transformations have on exponential functions?
- Which form of a linear equation is most appropriate to represent a given problem situation?

Lesson Title	Lesson Overview	Standards
Mathia	By the end of Module 2, Topic 1, students should complete the following units in Mathia:	
	Connecting Arithmetic Sequences and Linear Functions (3 workspaces)	
	Multiple Representations of Linear Functions (2 workspaces)	
	Transforming Linear Functions (4 workspaces)	
	Comparing Linear Functions in Different Forms (1 workspace)	



Connecting the Dots	This lesson builds from what students know about arithmetic sequences to a general understanding of linear functions. Students connect an arithmetic sequence written in explicit form to a linear function in slope-intercept form. They compare the terms of each equation and prove the common difference and slope are always constant and equal. First differences method is defined as a strategy to determine whether a table represents a linear relationship. Average rate of change is defined and presented graphically. Finally, students use what they know about arithmetic sequences to complete a graphic organizer to summarize the characteristics and representations of linear functions.	F.IF.A.1 F.IF.A.3 F.IF.B.6 F.LE.A.1a F.LE.A.1b F.LE.A.2
Fun with Functions, Linear Ones	Students determine whether functions represented as scenarios, equations, or graphs are linear functions. They extend what they learned about first differences to analyze tables with input values that are not consecutive integers. Students then analyze a scenario and graph that can be represented by a function in the form $f(x) = ax$ . A new scenario requires an equation of the form $f(x) = a(x - c)$ . They analyze the meaning of this shift in the graph in terms of the context and compare the structure to that of $f(x) = ax + b$ . The scenario changes a second time, and the students explore an equation in the form $f(x) = a(x - c) + d$ .	A.CED.A.1 A.REI.A.1 A.SSE.A.1a F.BF.B.3 F.IF.A.2 F.IF.B.4 N.Q.A.1
Get Your Move On	Students identify key characteristics of several linear functions. A graph and a table of values for the basic linear function $f(x) = x$ are given, and students investigate $f(x) + D$ and $f(x) = x$ are given, and students investigate $f(x) + D$ and $f(x) = x$ are given, and describe each transformation on $f(x)$ to produce $f(x) = x$ are given, and describe each transformation on $f(x) = x$ to produce $f(x) = x$ are given, and describe each transformation on $f(x) = x$ to produce $f(x) = x$ are given, and describe each transformation on $f(x) = x$ to produce $f(x) = x$ are given, and describe each transformation on $f(x) = x$ to produce $f(x) = x$ to produce $f(x) = x$ and $f(x) = x$ to produce $f(x) = x$ to produ	F.BF.B.3 F.IF.B.4 F.IF.C.7a
Comparing Linear Functions in Different Forms	Students compare linear functions represented in different forms to answer questions about real-world scenarios. They also identify the scale and origin on the graph of a function given a situation description. Finally, students generate and compare their own linear functions using tables, graphs, and equations.	F.IF.C.9 N.Q.A.1



# **Module 2 Topic 2: Solving Linear Equations and Inequalities**

Primary Resource: High School Math Solution Algebra I, Carnegie Learning, 2018.

## **Enduring Understandings**

- Relationships can be described, and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways.
- Mathematical situations and structures can be translated and represented abstractly using variables, expressions, and equations.
- If two quantities vary proportionally, that relationship can be represented as a linear function.
- Mathematical rules (relations) can be used to assign members of one set to members of another set. A special rule (function) assigns each member of one set to a unique member of the other set.
- Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities so solutions can be found.

#### **Essential Questions**

• How can we use equations and inequalities to solve problems?

Lesson Title	Lesson Overview	Standards
Mathia	By the end of Module 2, Topic 2, students should complete the following units in Mathia:  • Solving Linear and Literal Equations (4 workspaces)  • Modeling Linear Inequalities (3 workspaces)	
Striking a Balance	Students are given a mathematical sentence that is always true and one that is always false. They choose any variable or constraint and use the Properties of Equality to investigate ways to change the outcome of the given number sentence. Students reason that the mathematical sentence that is always true is still always true and that one that is false is still always false. The terms no solution and infinite solutions are defined. Finally, students play Tic-Tac-Bingo as they work together to create equations with given solution types from assigned expressions.	A.CED.A.1 A.REI.A.1 A.REI.B.3



It's Literally about Literal Equations	Students identify the slope and intercepts of functions in general, factored, and standard form. They determine the characteristics for the equations $Ax + By = C$ . They then explain which form is more efficient in determining the slope and the $x$ - and $y$ -intercepts. Next, the term literal equation is defined. Students rewrite different literal equations to solve for given variables.	A.CED.A.4 N.Q.A.1
Not All Statements are Made Equal	Students use the graph of a function modeling a scenario with a positive rate of change to determine solutions to linear inequalities. The term "solve an inequality" is defined. Students write and solve two-step inequalities algebraically, choosing the most accurate solution in the context of the problem situation. Students solve linear inequalities for a scenario with a negative rate of change that affects the sign of the inequality. Finally, they solve linear inequalities that require more than two steps to solve.	A.CED.A.1 A.CED.A.3 A.REI.B.3 N.Q.A.3
Don't Confound Your Compounds	The term compound inequality is defined. Students determine the inequality symbols that complete statements about a scenario represented by compound inequalities and express them in compact form. Given a scenario, they express the inequalities using symbols, then solve and graph the inequalities. The terms solution of a compound inequality, conjunction, and disjunction are defined. Students solve and graph compound inequalities, including those written in compact form.	A.CED.A.1 A.REI.B.3 N.Q.A.3



## **Module 2 Topic 3: Systems of Equations and Inequalities**

Primary Resource: High School Math Solution Algebra I, Carnegie Learning, 2018.

## **Enduring Understandings**

- Relationships can be described, and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways.
- Mathematical situations and structures can be translated and represented abstractly using variables, expressions, and equations.
- If two quantities vary proportionally, that relationship can be represented as a linear function.
- Mathematical rules (relations) can be used to assign members of one set to members of another set. A special rule (function) assigns each member of one set to a unique member of the other set.
- Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities so solutions can be found.

#### **Essential Questions**

- Which method is most efficient for solving a given system of equations or inequalities?
- How do you determine whether a system of equations is consistent or inconsistent?

Lesson Title	Lesson Overview	Standards
	By the end of Module 2, Topic 3, students should complete the following units in Mathia:	
	Introduction to Systems of Linear Functions (2 workspaces)	
Mathia	Using Linear Combinations to Solve a System of Linear Equations (2 workspaces)	
	Graphing Linear Inequalities in Two Variables (2 workspaces)	
	Graphing a System of Linear Inequalities (2 workspaces)	



Double the Fun	Students explore a scenario that can be modeled with a system of linear equations in standard form. They graph the equations using the intercepts. They determine the intersection of the lines graphically and algebraically using substitution. Finally, students write a system of equations for given scenarios and analyze the slopes and <i>y</i> -intercepts and their relevance to the problem situation. They solve each system of equations graphically and algebraically, concluding that for any system there is no solution, one solution, or an infinite number of solutions.	A.CED.A.2 A.REI.C.6 A.REI.D.10 A.REI.D.11 F.IF.C.7a
The Elimination Round	Students explore a system of equations with opposite <i>y</i> -coefficients that is solved for <i>x</i> by adding the equations together. The term linear combinations method is defined, and students analyze systems that are solved by multiplying either one or both equations by a constant to rewrite the system with a single variable. Students analyze different systems of equations to determine how they would rewrite the equations to solve for one variable. Next, they apply the linear combinations method to two real-world problems, one with fractional coefficients.	A.CED.A.2 A.REI.C.5 A.REI.C.6
Throwing Shade	Scenarios are used that are represented by two-variable inequalities. Students write the inequality, complete a table of values, and use the table of values to graph the situation. The terms half-plane and boundary line are defined. Students use shading and solid or dashed lines to indicate which regions on the coordinate plane represent solution sets to the problem situation. Multiple representations such as equations, tables, and graphs are used to represent inequalities and their solutions.	A.CED.A.2 A.CED.A.3 A.REI.D.12
Working the Constraints	The term <i>constraints</i> is defined. Students write a system of linear inequalities to model a scenario, and graph the system, determining that overlapping shaded regions identify the possible solutions to the system. They practice graphing several systems of inequalities determining the solution set. Finally, students match systems, graphs, and possible solutions to systems.	A.CED.A.2 A.CED.A.3 A.REI.D.12



Working the System	Students write a system of linear equations for each of three different scenarios: one in the form $y = ax + b$ , one in the form $y = a(x - c) + b$ , and one in the form $y = a(cx) + b$ . They use any method to solve the system before reasoning about the solution in terms of the problem context. Students write a system composed of four linear inequalities to model a scenario and graph the system. Students determine the correct region that contains the solution set that satisfies all the inequalities in the system.	A.CED.A A.REI.D A.REI.C
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.A.3 D.12 C.6



## **Module 3 Topic 1: Introduction to Exponential Functions**

Primary Resource: High School Math Solution Algebra I, Carnegie Learning, 2018.

## **Enduring Understandings**

- Relationships can be described, and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways.
- Mathematical situations and structures can be translated and represented abstractly using variables, expressions, and equations.
- If two quantities vary proportionally, that relationship can be represented as a linear function.
- Mathematical rules (relations) can be used to assign members of one set to members of another set. A special rule (function) assigns each member of one set to a unique member of the other set.
- Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities so solutions can be found.
- Objects in space can be transformed in an infinite number of ways, and those transformations can be described, and analyzed mathematically.

#### **Essential Questions**

- What is the relationship between arithmetic sequences and linear functions and geometric sequences and exponential functions?
- What are the effects that horizontal, vertical, and reflective transformations have on exponential functions?

Lesson Title	Lesson Overview	Standards
	By the end of Module 3, Topic 1, students should complete the following units in Mathia:	
Mathia	Geometric Sequences and Exponential Functions (1 workspace)	
	Transformations of Exponential Functions (6 workspaces)	



A Constant Ratio	Students learn through investigation that while all geometric sequences are functions, only some geometric sequences can be represented as exponential functions. They identify the constant ratio in different representations of exponential functions and then show algebraically that the constant ratio between output values of an exponential function is represented by the variable $b$ in the function form $f(x) = ab^x$ . Students also identify the $a$ -value of that form as the $y$ -intercept of the graph of the function. They learn to write an exponential equation from two given points. The lesson concludes with a comparison of the base of the power in the equation $f(x) = ab^x$ , the expression $(f(x + 1))/(f(x))$ , and the common ratio of the corresponding geometric sequence.	A.REI.D.10 F.BF.A.1a F.LE.A.1a F.LE.A.2 F.LE.B.5
The Power Within	Students explore a scenario that can be represented by the function $f(x) = 2^x$ . They use the rules of exponents and their understanding of a constant ratio to determine output values for the exponential function when the input values are non-integers. Students use this exploration to connect expressions with rational exponents to those in radical notation. Students learn the term horizontal asymptote and explore this concept on different graphs, analyzing end behavior, particularly as the x-values approach negative infinity. Finally, students practice converting between expressions with rational exponents and those in radical notation and make generalizations about the constant ratio for exponential functions.  NOTE – Students do not need to solve exponential equations, but they DO need to simplify square roots.	A.CED.A.1 A.REI.B.3 F.IF.C.8b F.LE.A.2 N.RN.A.1 N.RN.A.2 N.RN.B.3
Now I Know my A, B, C, Ds	Students explore a variety of different transformations of exponential functions, including vertical translations, horizontal translations, vertical reflections and dilations, and horizontal reflections and dilations. For each transformation, students sketch graphs of the transformation, compare characteristics of the transformed graphs with the graph of the parent function, including the horizontal asymptote when appropriate, and write transformations using coordinate notation. They also consider different ways to rewrite and interpret equations of function transformations. Finally, students summarize the effects of different transformations at the end of the lesson.	F.BF.B.3 F.IF.B.4 F.IF.C.7e F.IF.C.8b



## **Module 3 Topic 2: Using Exponential Functions**

Primary Resource: High School Math Solution Algebra I, Carnegie Learning, 2018.

## **Enduring Understandings**

- Mathematical situations and structures can be translated and represented abstractly using variables, expressions, and equations.
- Mathematical rules (relations) can be used to assign members of one set to members of another set. A special rule (function) assigns each member of one set to a unique member of the other set.
- Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities so solutions can be found.

## **Essential Questions**

- What is the relationship between linear functions and exponential functions?
- What are the effects that horizontal, vertical, and reflective transformations have on exponential functions?

Lesson Title	Lesson Overview	Standards
Mathia	By the end of Module 3, Topic 2, students should complete the following units in Mathia:  • Exponential Equations for Growth and Decay (4 workspaces)  • Modeling Using Exponential Functions (2 workspaces)	
Uptown and Downtown	Students compare linear and exponential functions in the context of simple interest and compound interest situations. They identify the values in the exponential function equation that indicate whether an exponential function is a growth or decay function, and they apply this reasoning in context.	A.CED.A.1 A.CED.A.2 A.REI.D.10 A.REI.D.11 A.SSE.B.3c F.IF.C.8b F.LE.B.5



Powers and the Horizontal Line	Students match exponential equations to their graphs to discern that the horizontal asymptote is always represented by $y = D$ . For exponential growth and decay scenarios, students complete tables of values, graph the functions, and write exponential equations using function notation. Students use graphs to estimate the solutions to equations by graphing both sides of the equation and locating the point of intersection. They use the properties of exponents to rewrite the $b$ and $b$ -values of exponential functions in equivalent forms to reveal properties of he quantity represented in the function. This allows them to reinterpret an equation showing annual rates of increase for a mutual fund to show monthly and quarterly rates of increase for the same fund.	A.CED.A.1 A.CED.A.2 A.REI.D.10 A.REI.D.11 A.SSE.B.3c F.IF.C.8b F.LE.B.5
Savings, Tea, and Carbon Dioxide	Students model two savings scenarios, one by means of an exponential function $f(x)$ and one by a constant function $g(x)$ . Then they create a third function $h(x) = f(x) + g(x)$ , graph all three functions on the same graph, and explain how they are related. Given a data set, students create a scatter plot, write a regression equation, use the function to make predictions, and interpret the reasonableness of a prediction. The lesson concludes with students generalizing about common features of scenarios that are modeled by exponential functions. They also describe the shape of a scatter plot representing an exponential function and sketch possible graphs of exponential functions.	F.BF.A.1b N.Q.A.2 S.ID.B.6a



## **Module 5 Topic 1: Introduction to Quadratic Equations**

Primary Resource: High School Math Solution Algebra I, Carnegie Learning, 2018.

## **Enduring Understandings**

- Mathematical situations and structures can be translated and represented abstractly using variables, expressions, and equations.
- Mathematical rules (relations) can be used to assign members of one set to members of another set. A special rule (function) assigns each member of one set to a unique member of the other set.
- Objects in space can be transformed in an infinite number of ways, and those transformations can be described, and analyzed mathematically.

#### **Essential Questions**

• How are domain, range, intervals of increase and decrease, absolute maximum or absolute minimum, symmetry, zeros, and graphical behavior determined for quadratic functions?

Lesson Title	Lesson Overview	Standards
Mathia	<ul> <li>By the end of Module 5, Topic 1, students should complete the following units in Mathia:</li> <li>Exploring Quadratic Functions (6 workspaces)</li> <li>Key Characteristics of Quadratic Functions (2 workspaces)</li> <li>Transformations of Quadratic Functions (5 workspaces)</li> <li>Sketching and Comparing Quadratic Functions (3 workspaces)</li> </ul>	
Up and Down or Down and Up	Students are introduced to quadratic functions and their growth pattern through a sequence of pennies. They are then provided four different contexts that can be modeled by quadratic functions. For each function, students address the key characteristics of the graphs and interpret them in terms of the context. They also compare the domain and range of the functions and the context they represent. The first context involves area and is used to compare and contrast linear and quadratic relationships. The second context involves handshakes and has the student write the function. The third context involves catapulting a pumpkin. Students analyze this function written in general form. The final context involves revenue and demonstrates how a quadratic function can be written as the product of two linear functions.	A.REI.D.10 A.REI.D.11 F.IF.B.4 F.IF.B.5 F.IF.C.7a



Endless Forms Most Beautiful	Students revisit the four scenarios from the previous lesson to introduce equivalent quadratic equations with different structures to reveal different characteristics of their graphs. They learn that a table of values represents a quadratic function if its second differences are constant. Students analyze the effect of the leading coefficient on whether the parabola opens up or down. They identify the axis of symmetry and vertex for each of the graphs using the equations in each form. Finally, students determine the <i>x</i> - and <i>y</i> -intercepts along with intervals of increase and decrease, using a combination of technology, symmetry, and equations.	A.APR.B.3 A.SSE.A.1a A.SSE.B.3a F.IF.B.4 F.IF.B.6 F.IF.C.7a F.IF.C.8a
More than Mets the Eye	Students explore a variety of different transformations of quadratic functions, including vertical translations, horizontal translations, vertical dilations and reflections, and horizontal dilations and reflections. For each transformation, students sketch graphs of the transformation, compare characteristics of the transformed graphs with those of the graph of the basic function, and write the transformation using coordinate notation. Students write quadratic equations in vertex form using the coordinates of the vertex and another point on the graph and in factored form using the zeros and another point on the graph.	A.SSE.B.3a F.BF.B.3
You Lose Some, You Lose Some	Students compare quadratic functions in standard form, factored form, and vertex form, then analyze the properties of each form. Students then answer questions to compare linear, quadratic, and exponential functions. They compute average rates of change for the functions across different intervals and then compare the change in the average rates of change across the different intervals. Quadratic equations in different forms are compared by identifying key characteristics of their representations.	A.CED.A.4 A.SSE.A.1a F.IF.C.9 F.LE.A.3



# **Module 5 Topic 2: Solving Quadratic Equations**

Primary Resource: High School Math Solution Algebra I, Carnegie Learning, 2018.

## **Enduring Understandings**

- Mathematical situations and structures can be translated and represented abstractly using variables, expressions, and equations.
- Mathematical rules (relations) can be used to assign members of one set to members of another set. A special rule (function) assigns each member of one set to a unique member of the other set.
- Objects in space can be transformed in an infinite number of ways, and those transformations can be described, and analyzed mathematically.

#### **Essential Questions**

- How are polynomial expressions simplified?
- How are graphing and factoring used to solve quadratic equations?

Lesson Title	Lesson Overview	Standards
	By the end of Module 5, Topic 2, students should complete the following units in Mathia:	
	Adding, Subtracting, and Multiplying Polynomials (7 workspaces)	
Mathia	Representing Solutions to Quadratic Equations (2 workspaces)	
	Factoring and Completing the Square (8 workspaces)	
	The Quadratic Formula (2 workspaces)	
This Time with Polynomials	Students are introduced to polynomials and identify the terms and coefficients of polynomials. Students sort polynomials by the number of terms, rewrite in general form if possible, and identify the degree. Students add and subtract polynomial functions algebraically and graphically and then determine that polynomials are closed under addition and subtraction. Students use area models and the Distributive Property to determine the product of binomials. They explore special products and are introduced to the terms difference of two squares and perfect square trinomial.	A.APR.A.1 A.SSE.A.1a



Solutions More or Less	Students use the Properties of Equality and square roots to solve simple quadratic equations. They express solutions in terms of the distance from the axis of symmetry to the parabola. Students identify double roots, estimate square roots, and extract perfect roots from the square roots of products. They show graphically that a quadratic function is the product of two linear functions with the same zeros. Students then use the Zero Product Property to explain that the zeros of a quadratic function are the same as the zeros of its linear factors. Finally, they rewrite any quadratic in the form $f(x) = ax^2 - c$ as the product of two linear factors.	A.REI.B.4b A.REI.D.10 A-SSE.A.2 A-SSE.B.3a N.RN.A.2
The Missing Link	Students recall how to factor out the GCF from different polynomials. They follow examples to factor quadratic trinomials, first using area models and then recognizing patterns in the coefficients. Students use the Zero Product Property to solve quadratic equations by factoring. They are then introduced to completing the square, a method they can use to convert a quadratic equation given in general form to vertex form. Students complete the square to solve quadratic equations that cannot be solved using other methods.	A.REI.B.4a A.SSE.B.3b F.IF.C.8b
Ladies and Gentlemen, the Quadratic Formula!	The first activity focuses on the graphical interpretation of $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ as the distance from $\left(\frac{-b}{2a}, 0\right)$ to each root. Students are introduced to the Quadratic Formula as a method to calculate the solutions to any quadratic equation written in general form. Students use the discriminant to determine the number and type of roots for a given function. Students learn why rational numbers are closed under addition and that the sum or product of a rational number and an irrational number is an irrational number. Students reason about the solution to a function with no <i>x</i> -intercepts. Students practice simplifying expressions with negative roots.	A.REI.B.4a A.REI.B.4b A.SSE.A.1b N.RN.A.2 N.RN.B.3



## **Module 5 Topic 3: Applications of Quadratics**

Primary Resource: High School Math Solution Algebra I, Carnegie Learning, 2018.

#### **Enduring Understandings**

- Mathematical situations and structures can be translated and represented abstractly using variables, expressions, and equations.
- Mathematical rules (relations) can be used to assign members of one set to members of another set. A special rule (function) assigns each member of one set to a unique member of the other set.
- Objects in space can be transformed in an infinite number of ways, and those transformations can be described, and analyzed mathematically.
- Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities so solutions can be found.

#### **Essential Questions**

- What strategies can be used to solve systems of non-linear equations?
- How do you choose the appropriate model to represent real-world data?
- How can real-world situations be modeled by quadratic functions to help solve problems?

Lesson Title	Lesson Overview	Standards
Mathia	By the end of Module 5, Topic 2, students should complete the following units in Mathia:  • Using Quadratic Functions to Model Data (1 workspace)	
All Systems Are Go!	Students are presented with a scenario that can be modeled with a quadratic and a linear equation and reason about the intersections of the two equations in the context of the problem. Next, they solve systems of equations composed of a linear equation and a quadratic equation algebraically using substitution, factoring, and the Quadratic Formula. They then verify the solutions graphically by determining the coordinates of the points of intersection. Finally, students solve a system composed of two quadratic equations using the same methods. They conclude that a system of equations consisting of a linear and a quadratic equation can have one solution, two solutions, or no solutions, while a system of two quadratic equations can have one solution, two solutions, no solutions, or infinite solutions.  NOTE – The focus on solving quadratic/linear systems should be both graphically and algebraically.	A.CED.A.2 A.CED.A.3 A.REI.C.7 A.REI.D.11



	make predictions using the regression equation. Students then analyze another data set and determine the	S.ID.B.6a F.BF.B.4a F.BF.B.4d F.IF.C.7b
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## **Module 2 Topic 4: Functions Derived from Linear Relationships**

Primary Resource: High School Math Solution Algebra I, Carnegie Learning, 2018.

## **Enduring Understandings**

- Relationships can be described, and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways.
- Mathematical situations and structures can be translated and represented abstractly using variables, expressions, and equations.
- If two quantities vary proportionally, that relationship can be represented as a linear function.
- Mathematical rules (relations) can be used to assign members of one set to members of another set. A special rule (function) assigns each member of one set to a unique member of the other set.
- Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities so solutions can be found.

#### **Essential Questions**

- What characteristics do linear absolute value functions have?
- How do absolute value functions, piecewise functions, and their graphs model real-world problems?
- How does the table of values of a function relate to the table of values of its inverse?

Lesson Title	Lesson Overview	Standards
Mathia	<ul> <li>By the end of Module 2, Topic 4, students should complete the following units in Mathia:</li> <li>Defining Absolute Value Functions and Transformations (5 workspaces)</li> <li>Absolute Value Equations and Inequalities (5 workspaces)</li> </ul>	
Putting the V in Absolute Value	Students model absolute value functions and their transformations on a human coordinate plane. They explore and analyze different transformations of absolute value functions, their graphs, and equations, and summarize the effects of these transformations. By transforming absolute value functions, students distinguish between the effects of changing values inside the argument of the function vs. changing values outside the function.  NOTE – Students can review transformations of quadratic functions in this lesson.	F.BF.B.3 F.IF.C.7b



Don't Confound Your Compounds	The term compound inequality is defined. Students determine the inequality symbols that complete statements about a scenario represented by compound inequalities and express them in compact form. Given a scenario, they express the inequalities using symbols, then solve and graph the inequalities. The terms solution of a compound inequality, conjunction, and disjunction are defined. Students solve and graph compound inequalities, including those written in compact form.	A.CED.A.1 A.REI.B.3 N.Q.A.3
Play Ball	Students create a linear absolute value function to model a scenario and use the graph of the function to estimate solutions in the context of the problem situation. They then solve a linear absolute value equation algebraically by first rewriting it as two separate linear equations. One equation represents the case where the value of the expression inside the absolute value is positive, and the second represents the case where it is negative. Students model a scenario with a linear absolute value inequality and its corresponding graph. Finally, they solve linear absolute value inequalities algebraically by first rewriting them as equivalent compound inequalities.	A.CED.A.2 A.CED.A.3 A.REI.D.11 F.IF.C.7b
I Graph in Pieces	Students develop a piecewise function from a scenario. The terms piecewise function and linear piecewise function are defined. They analyze a piecewise graph and write a scenario and piecewise function to represent the graph. Students analyze statements that correspond to different pieces of the graphed function. They then write a scenario that can be modeled with a piecewise function, graph a partner's scenario, and work with their partner to write a piecewise function for each.	F.IF.B.4 F.IF.C.7b
Step by Step	Students analyze the graph of a special piecewise function. The terms discontinuous graph and step function are defined, and students interpret those definitions through examination of graphs. Students are then given a context and must provide both the piecewise function and the graph that models it, explaining why the function is a step function. Next, they are introduced to the greatest integer function (floor function) and least integer function (ceiling function) through their definitions, notation, meaning of individual values, graphs, and real-world examples.	F.IF.C.7b



A Riddle Wrapped in a Mystery

Students use a table of values to determine the inverse of a given problem situation. The term inverse of a function is defined, and students are shown how to algebraically determine the inverse of a function. They then create the graph of the inverse of a linear function by reflecting the original function across the line y = x using patty paper. The term one-to-one function is defined, and students determine whether given functions are one-to-one functions.

F.BF.B.4a



# **Module 4 Topic 1: One-Variable Statistics**

Primary Resource: High School Math Solution Algebra I, Carnegie Learning, 2018.

## **Enduring Understandings**

- Data can be represented visually using tables, charts, and graphs. The type of data determines the best choice of visual representation.
- There are special numerical measures that describe the center and spread of numerical data sets.

#### **Essential Questions**

- What measures describe the center of a set of data?
- What measures describe the spread of a set of data?
- What are the best ways to display a set of data?
- How can one-variable statistics be used to solve problems?

Lesson Title	Lesson Overview	Standards
Mathia	By the end of Module 4, Topic 1, students should complete the following units in Mathia:  • Defining Absolute Value Functions and Transformations (5 workspaces)  • Absolute Value Equations and Inequalities (5 workspaces)	
Way to Represent	The statistical process is reviewed. Students analyze a small data set given by creating a dot plot. Next, a much larger data set for the same scenario is presented in a frequency table. Students construct and analyze a histogram for the data. A worked example shows how to use a five-number summary to create a box-and-whisker plot. Students are given two five-number summaries of data comparing the same variable and use them to construct and analyze two box-and-whisker plots. They then write an analysis comparing the two data sets.	S.ID.A.1 S.ID.A.2



A Skewed Reality	Students are presented with various data displays and predict the location of the mean and median in each display. A worked example presents the formula for calculating the arithmetic mean and introduces students to the formal notation. Students construct a box-and-whisker plot that overlays a given dot plot to analyze the spread of the data points. The term interquartile range (IQR) is introduced, and students calculate the IQR for the same data set. They remove any outliers and reanalyze the IQR of the data set. Next, students compare two new data sets displayed in a table and in box-and-whisker plots, removing possible outliers. They then calculate and interpret the standard deviation to compare three symmetric data sets. At the end of the lesson, students know when and how to use mean and standard deviation vs. mean and IQR to describe the center and spread of a data set.	S.ID.A.1 S.ID.A.2 S.ID.A.3
Dare to Compare	Students conclude that when comparing two data sets, if one data set is skewed, then the median and IQR should be used to compare the sets. Next, students are provided with three scenarios that each compare two different data sets. In the each, students are provided with a table comparing two data sets and must decide which measure of center and spread to use in their comparison.	S.ID.A.1 S.ID.A.2 S.ID.A.3



## **Module 4 Topic 2: Two-Variable Categorical Data**

Primary Resource: High School Math Solution Algebra I, Carnegie Learning, 2018.

## **Enduring Understandings**

- Some questions can be answered by collecting and analyzing data, and the question to be answered determines the data that needs to be collected and how best to collect it.
- Data can be represented visually using tables, charts, and graphs. The type of data determines the best choice of visual representation.
- There are special numerical measures that describe the center and spread of numerical data sets.

#### **Essential Questions**

- How can two-way frequency tables be used to summarize categorical data?
- How do joint, marginal, and conditional relative frequencies differ?
- How can joint, marginal, and conditional relative frequencies be calculated?
- How can two-variable data sets be used to solve real-world problems?

Lesson Title	Lesson Overview	Standards
It Takes Two	Students differentiate between questions that are answered with numeric data from those answered with categorical data. They are presented with data expressed as categories rather than numerical values. The terms two-way frequency table, frequency distribution, joint frequency, and marginal frequency distribution are defined. Students organize data into a two-way frequency table and create a marginal frequency distribution and bar graphs to answer questions related to the given scenario. Finally, students interpret the data analyzed in the context of the scenario.	S.ID.B.5
Relatively Speaking	Students construct a relative frequency distribution and marginal relative frequency distribution using data for a scenario. They analyze the distributions and answer questions about the problem situation. Next, students are shown stacked bar graphs that represent the relative frequency distribution in two different ways. They compare the graphs to the tables in the previous activity and explain the advantages of graphing the data each way. Finally, students analyze and interpret the data represented by the stacked bar graphs in terms of the problem situation.	S.ID.B.5



On One Condition or More	Students consider what different joint frequencies in a marginal relative frequency distribution represent. They construct a stacked bar graph and analyze the percentages shown in the graph before the term conditional relative frequency distribution is introduced. Students construct a conditional relative frequency distribution and use it to answer questions related to the given scenario. They construct a second conditional relative frequency distribution in terms of the other variable. Finally, students construct a conditional relative frequency distribution and interpret the data in terms of the problem situation.	S.ID.B.5
Data Jam	Students synthesize what they know about analyzing and interpreting two-variable categorical data to make a recommendation in a real-world scenario. They organize a given data set by creating a frequency distribution and a stacked bar graph, and then use conditional relative frequency distributions to determine whether there is an association between the two categories. Students formulate conclusions for specified subsets of the data and use statistics to support their conclusions.	S.ID.B.5